K2 MICROLENSING: PROJECTED EARTH-SATLLITE SEPARATION

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Abstract: We investigate how K2's changing projected separation during its proposed microlensing campaign, from 0.1 AU to 0.8 AU, impacts its ability to return high-impact science. We find that K2's separations are almost optimal for free floating planets (the main science driver) and very good for main-sequence stars, particularly for main-sequence hosts of planets.

Key words: planets – gravitational microlensing

1. INTRODUCTION

In our previous white paper, we argued broadly that efficient microlens parallaxes require a projected separation between two simultaneous observers of roughly 1 AU, and therefore require that the second observer be a satellite in solar orbit. During the available observing window (roughly 7 Apr – 29 Jun 2016) K2 will actually vary from a minimum of about 0.1 AU to a maximum of about 0.8 AU. Here we therefore address the question of how this finite interval affects K2 science.

2. REVIEW OF RELEVANT EQUATIONS

Most microlensing events are fully characterized by just 3 parameters, t_0 (time of peak), u_0 (impact parameter in units or the "Einstein radius" $\theta_{\rm E}$), and $t_{\rm E}$ (Einstein timescale),

$$t_{\rm E} = \frac{\theta_{\rm E}}{\mu}; \quad \theta_{\rm E}^2 = \kappa M \pi_{\rm rel}; \quad \kappa = \frac{4G}{c^2 {\rm AU}} = 8.1\,\frac{\rm mas}{M_\odot}. \quad (1)$$

Here $\pi_{\rm rel}$ and μ are the lens-source relative parallax and proper motion, respectively, and M is the lens mass. Thus, M, $\pi_{\rm rel}$ and μ are generally not known separately, but only through the peculiar combination of them in the observable $t_{\rm E}$.

The goal of a K2 microlensing mission is to measure the "microlens parallax" $\boldsymbol{\pi}_{\rm E}$,

$$\boldsymbol{\pi}_{\rm E} = \pi_{\rm E}\frac{\boldsymbol{\mu}}{\mu}; \qquad \pi_{\rm E} = \frac{\pi_{\rm rel}}{\theta_{\rm E}} \quad (2)$$

If this quantity is measured, then (together with $\theta_{\rm E}$, which is routinely measured in planetary microlensing events) it yields

$$M = \frac{\theta_{\rm E}}{\kappa \pi_{\rm E}}; \qquad \pi_{\rm rel} = \theta_{\rm E}\pi_{\rm E}. \quad (3)$$

Then, since the source parallax π_s is usually quite well known, the lens distance can be determined $D_l = {\rm AU}/(\pi_{\rm rel} + \pi_s)$.

If the observer changes position by a vector distance $\Delta\mathbf{x}$, then the apparent separation of the lens and source will change by $\Delta\boldsymbol{\theta} = (\pi_{\rm rel}/{\rm AU})\Delta\mathbf{x}$. Hence, the separation in the Einstein ring will change by

$$\Delta\mathbf{u} = \frac{\Delta\boldsymbol{\theta}}{\theta_{\rm E}} = \frac{\pi_{\rm rel}}{\theta_{\rm E}}\frac{\Delta\mathbf{x}}{\rm AU} = \pi_{\rm E}\frac{\Delta\mathbf{x}}{\rm AU} \quad (4)$$

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Because a displacement $\Delta\mathbf{u}$ in the Einstein ring leads to measurable changes in magnification, and since $\Delta\mathbf{x}$ is of course known, such displaced observations can yield a parallax measurement.

Note that even if $\theta_{\rm E}$ is not measured, it can be estimated to factor 1.5 accuracy ($1\,\sigma$) from the fact that almost all microlensing events have roughly the same lens-source relative proper motion, $\mu_{\rm typical} \sim 4\,{\rm mas\,yr^{-1}}$, to within a factor 1.5 ($1\,\sigma$). Then we can estimate $\theta_{\rm E,estimated} = \mu_{\rm typical} t_{\rm E,observed}$.

3. RULE OF THUMB

The "rule of thumb" for obtaining credible parallax information is

$$0.03 \lesssim \pi_{\rm E}\frac{\Delta x}{\rm AU} \lesssim 1 \quad (5)$$

This equation invites two questions. First, how are the boundaries established? Second how well does K2 satisfy this equation for "typical events"?

The reason for the upper boundary is simple. The microlensing magnification A is given (for point lenses) by

$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}, \qquad u(t) = \sqrt{u_0^2 + \frac{(t-t_0)^2}{t_{\rm E}^2}} \quad (6)$$

Hence, this magnification dies off very quickly for $u > 1$, i.e. $A(1, 1.5, 2, 2.5, 3) = (1.34, 1.13, 1.06, 1.03, 1.017)$ Hence, if an event is seen from the Earth, then $u_{0,\oplus} \lesssim 1$ and so if $\pi_{\rm E}\Delta x \gtrsim 1\,{\rm AU}$, then most likely $u_{0,\rm sat} \gtrsim 1$ and hence it most likely will not be seen. Of course, this boundary is not strict. First, if the trajectory happens to be well aligned with the Earth-satellite separation vector, then both Earth and the satellite will experience well characterized events even though they are separated by over an Einstein radius. Second, for bright sources (so excellent photometry), even very low-amplitude events may be detected (particularly because one knows to look for them from the presence of an event in the other observatory (either Earth or satellite).

The other boundary is more approximate. For typical events, it is possible to measure u_0 and t_0 to 1% and $0.01\,t_{\rm E}$ precision respectively from ground-based data. Hence, if the difference in these parameters is 3% and/or $0.03\,t_{\rm E}$ (and assuming that the continuous satellite observations will yield better data than the ground-based data) then this will yield $3\,\sigma$ measurements. In many cases, the situation is substantially more favorable. For example if there are caustic crossings, these can usually be measured to precisions 10–100 times better. But there are also less favorable cases, either high-u_0 events or those with poor signal-to-noise photometry. Finally, of potential relevance (but actually turning out not to be very important) is the fact that for short events such as those caused by free-floating planets (FFPs), the precision is often worse, simply because there are fewer data points.

4. Viability of K2 Separations

Of course, real microlensing events are drawn from a large continuous range of parameters. However, to understand the viability of the K2 range of separations, we should consider a few typical cases. Hence, we write down the *same* equation for $\pi_{\rm E}$ but with four different normalizations. These relate to "typical hosts" $M = 0.5\,M_\odot$ and "typical FFPs" $M = 1\,M_{\rm jup}$, and to "typical disk lenses" $\pi_{\rm rel} = 125\,\mu$as and "typical bulge lenses" $\pi_{\rm rel} = 10\,\mu$as.

$$\pi_{\rm E} = 4.00\sqrt{\frac{\pi_{\rm rel}/125\,\mu{\rm as}}{M/M_{\rm jup}}} = 1.13\sqrt{\frac{\pi_{\rm rel}/10\,\mu{\rm as}}{M/M_{\rm jup}}} \qquad (7)$$

$$\pi_{\rm E} = 0.18\sqrt{\frac{\pi_{\rm rel}/125\,\mu{\rm as}}{M/0.5\,M_\odot}} = 0.05\sqrt{\frac{\pi_{\rm rel}/10\,\mu{\rm as}}{M/0.5\,M_\odot}} \qquad (8)$$

The first point to consider is that there is no "one size fits all". Even these central values of different "typical" populations span a range of $4.00/0.05 = 80$. This is already larger than the factor ~ 30 range of the "rule of thumb" Equation (5). Thus, it is impossible to fully capture these diverse phenomena from a single Earth-satellite separation. To get a more concrete sense of this, we evaluate the range of $\pi_{\rm E}\Delta x$ for each of the above four examples separately, using the K2 range $0.1 < \Delta x/{\rm AU} < 0.8$

$$0.4 < \pi_{\rm E}\frac{\Delta x}{\rm AU} < 3.2, \qquad (\text{"disk FFP"}) \qquad (9)$$

$$0.11 < \pi_{\rm E}\frac{\Delta x}{\rm AU} < 1.0, \qquad (\text{"bulge FFP"}) \qquad (10)$$

$$0.02 < \pi_{\rm E}\frac{\Delta x}{\rm AU} < 0.14, \qquad (\text{"disk star"}) \qquad (11)$$

$$0.005 < \pi_{\rm E}\frac{\Delta x}{\rm AU} < 0.04 \qquad (\text{"bulge star"}) \qquad (12)$$

The principal science driver for K2 is FFPs. Therefore we should begin by analyzing these. The situation is best for bulge FFPs. For these, the anticipated range of $\pi_{\rm E}\Delta x/{\rm AU}$ (0.11–1.0) is entirely contained within the allowed range (from Equation (5)) (0.03 – 1.0). For disk FFPs, the anticipated range (0.4–3.2) falls outside the allowed range for relatively large Earth-satellite projected separations $\Delta x > 0.25$AU. As mentioned above, some of these will be recovered because the trajectory direction is favorable. In other cases, there will only be a lower limit on $\pi_{\rm E}$. This will be adequate to confirm them as FFPs, but not to actually measure their mass. Finally note that because FFPs generally avoid the lower boundary of $\pi_{\rm E}\Delta x > 0.03\,$AU (set by smallness of the effect requiring high-precision photometry), there is never any real problem due to the short timescales degrading parameter measurement.

At the opposite extreme, bulge stellar lenses really require large Earth-satellite separations. For these lenses, and for times when K2 has a short projected separation, one may only get an upper limit on $\pi_{\rm E}$ and so determine that it is a bulge lens, but not make a precise mass measurement. However, it should be stressed that from the standpoint of K2-microlensing science drivers, the most interesting bulge stellar lenses will have caustic crossings (due to binary or planetary companions) and for these, precise measurements are possible even at 10 or 100 times smaller $\pi_{\rm E}\Delta x$. Finally, we note that disk stellar lenses mostly fit into the "rule of thumb" range.

Another consideration is that the period of very short Earth-K2 separation opens the possibility of detecting Neptune-mass FFPs. These objects have $\pi_{\rm E}$ that are 4.6 times larger than Jupiter-mass FFPs:

$$\pi_{\rm E} = 20\sqrt{\frac{\pi_{\rm rel}/125\,\mu{\rm as}}{M/M_{\rm Nep}}} = 5.2\sqrt{\frac{\pi_{\rm rel}/10\,\mu{\rm as}}{M/M_{\rm Nep}}} \qquad (13)$$

Hence, for separations $\Delta x = 0.1\,$AU, the bulge FFP-Neptunes are clearly within range and the disk FFP-Neptunes are plausibly within range for those events with favorably oriented trajectories. Hence, the time that K2 is at short baselines provides important discovery potential for low-mass FFPs.

5. Conclusion

During the K2-microlensing campaign, K2 will be at a range of projected separations spanning a factor ~ 8. Because of the factor ~ 30 range in "acceptable" separations withing the Einstein ring (0.03–1 Einstein radii), this range of physical projected separations makes K2 sensitive to a huge range of phenomena, from Neptune-mass FFPs to main-sequence stars (including those with planets), i.e., a factor 10,000 in mass. It is true that not all phenomena can be probed at all separations, but the overlap between required separations and K2's actual projected separation from Earth is remarkably good, particularly for the highest-priority scientific targets.